Relativistic Transport with Many-body Interactions

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Particle productions involving three or more particles in the final state are import processes in studying relativistic heavy ion collisions. Proper treatment of these processes are essential in achieving reasonable equilibrium properties for matters at extreme temperatures and densities. To maintain detailed balance one must also include the inverse channels, i.e. interactions involving many-body initial states. One of a serious problems in most transport models is that these types of interactions are not taken into account. Almost all current event generators for RHIC either break detailed balance explicitly, or are limited to one-to-two and two-to-two interactions.

We developed a relativistic transport code, the General Cascade Program – GCP[1], which allows arbitrary many-body interactions. The code is an efficient solver of the relativistic Boltzmann equation using the cascade algorithm. The most general form of the Boltzmann equation with arbitrary n-to-m interactions is

$$p^\mu \partial_\mu \rho_a(\vec{x}, \vec{p}, t) = I$$

where ρ_a is probability density for particle specie a, and I on the right hand side stands for

$$\begin{aligned}
&\sum_n \sum_{b_1, b_2, \cdots, b_n} \int \prod_{i=1}^{n} \frac{d^3\vec{p}_{b_i}}{(2\pi)^3 2E_{b_i}} \rho_{b_i}(\vec{x}, \vec{p}_{b_i}, t) \\
&\cdot \sum_m \sum_{c_1, c_2, \cdots, c_m} \int \prod_{j=1}^{m} \frac{d^3\vec{p}_{c_j}}{(2\pi)^3 2E_{c_j}} \mid A_{n\to m} \mid^2 \\
&\cdot (2\pi)^4 \delta^4 (\sum_{i=1}^{n} p_{b_i} - \sum_{j=1}^{m} p_{c_j}) \\
&\cdot [-\sum_{i=1}^{n} \delta_{ab_i} \delta^3(\vec{p} - \vec{p}_{b_i}) + \sum_{j=1}^{m} \delta_{ac_j} \delta^3(\vec{p} - \vec{p}_{c_j})]
\end{aligned}$$

and $\mid A_{n\to m} \mid^2$ is given by the n-to-m-body S-matrix.

In the cascade algorithm, whenever two particles come within a distance of less than $\sqrt{\sigma/\pi}$, a two-body collision occurs. The two-body collision rate is than proportional to the product of two densities. This is consistent is with the equation above. We have now generalized this concept to n-body interactions. For a set of n particles, we check whether each particle i ($i = 1, 2, \cdots, n$) is headed towards the n-body c. of m. and also whether that particle will come within an interaction range R of the c. of m. Clearly, the n-body collision rate is proportional to the product of n densities.

As with two-body interactions, we can write down a set of Lorentz covariant formulae for the n-body interaction criterion. The particle i enters and leaves the interaction range R at time

$$t_{ci} = t_i + \frac{E_i(p_i p_{\rm cm})(C_i \pm H_i)}{(p_i p_{\rm cm})[(p_i p_{\rm cm})^2 - p_i^2 p_{\rm cm}^2]}.$$

where

$$\begin{aligned}
C_i &\equiv (x_i p_{\rm cm})(p_i p_{\rm cm}) - (x_i p_i) p_{\rm cm}^2 + F_i \\
D_i &\equiv (x_i p_{\rm cm})^2 - x_i^2 p_{\rm cm}^2 - 2G_i \\
&+ \frac{1}{p_{\rm cm}^2} \sum_i [(x_i p_{\rm cm}) F_i + (p_i p_{\rm cm}) G_i] - R^2 p_{\rm cm}^2 \\
F_i &\equiv \sum_j [(p_i x_j)(p_j p_{\rm cm}) - (p_i p_j)(x_j p_{\rm cm})] \\
G_i &\equiv \sum_j [(x_i p_j)(x_j p_{\rm cm}) - (x_i x_j)(p_j p_{\rm cm})], \\
H_i^2 &\equiv C_i^2 - \Big((p_i p_{\rm cm})^2 - p_i^2 p_{\rm cm}^2\Big) D_i
\end{aligned}$$

So the condition that particle i is headed towards the n-body c. of m. is $C_i < 0$, and that it will get within range R if and only if $H_i^2 > 0$.

References

[1] Y. Pang, *The General Cascade Program*, to be published.

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